COMPARISON STUDY OF NUMERICAL METHODS APPLIED FOR ESTIMATION OF HEAT TRANSFER COEFFICIENT DURING QUENCHING

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ABSTRACT

Better understanding the heat transfer between the surface of a workpiece and the cooling medium occurring during the immersion quenching process requires accurate estimation of Heat Transfer Coefficient (HTC) [1,2]. In this paper an inverse heat transfer formulation based on an iterative regularization algorithm is outlined which is developed for evaluation of HTC during heat treatment. The applicability of the inverse heat transfer methodology is demonstrated on an example.

1. THE INVERSE HEAT CONDUCTION PROBLEM

The temperature distribution inside a homogeneous isotropic domain (Ω) with constant material properties (Fig 1.) is governed by

\[ \nabla \cdot \left( k(r, T) \nabla T \right) + Q(r, T, t) = C_p(r, T) \rho(r, t) \frac{\partial T}{\partial t} \]

(1)

where \( r \) is the spatial vector and \( r \in \Omega \), \( t \) is the time, \( k \) is the heat conductivity, \( T \) is the temperature, \( C_p \) is the specific heat, \( \rho \) is the density and \( Q \) is the latent heat. The initial condition is

\[ T(r, t = 0) = T_0(r) \]

(2)

where \( T_0 \) is the initial temperature of the domain. The boundary conditions are expressed by

\[ -k \frac{\partial T}{\partial r} = h_i(T(r, t) - T_\text{am}) \quad \text{in} \Gamma_i, \quad i = 1 \ldots p \]

(3)

where \( h_i \) are the heat transfer coefficients corresponding to different portions of the boundary (\( \Gamma_1 \cup \Gamma_2 \ldots \cup \Gamma_p = \Gamma \) and \( \Gamma_1 \cap \Gamma_2 \ldots \cap \Gamma_p = \emptyset \)) and \( T_\text{am} \) is the ambient temperature. Each one
of these p boundary zones has a time dependent heat transfer coefficient to be optimized. The
time dependence of the heat transfer coefficient can be approximated by polygonal functions,
each one defined by a set of parameters \( h_{i}^{(r)} = (r=1, \ldots, p; i=1, \ldots, q) \), according to Fig 2.

![Figure 1. The representation of the domain](image1)

![Figure 2. Time approximations of HTC](image2)

The unknown design parameters can be expressed by the vector of \( m \) (\( m = p \times q \)), components \( \tau = (\tau_{1}, \ldots, \tau_{m}) = (h_{1}^{(1)}, \ldots, h_{q}^{(1)}, h_{1}^{(2)}, \ldots, h_{q}^{(2)}, \ldots, h_{1}^{(p)}, \ldots, h_{q}^{(p)}) \). The temperature at different
instants of time is given by measurements at \( n \) points in the solid region, located at \( r_{k} \),
\( (k=1, \ldots, n) \). On calling \( T_{m}^{k} \), the measured temperatures, and \( T_{c}^{k} \), the numerically calculated
temperature at those points, one can pose the problem of obtaining the values of the heat
transfer coefficients \( \tau_{i} \) that minimize the function:

\[
S = S(\tau_{1}, \ldots, \tau_{m}) = \sum_{k=1}^{n} (T_{m}^{k} - T_{c}^{k})^{2} = \min
\]

being \( n \) the total number of measured temperatures, i.e. the number of points times the
number of measurements at each point. A necessary condition to satisfy is that the following
set of equations must be verified simultaneously (4):

\[
F_{i} = \frac{\partial S}{\partial \tau_{i}} = -2 \sum_{k=1}^{n} (T_{m}^{k} - T_{c}^{k}) \frac{\partial T_{c}^{k}}{\partial \tau_{i}} = 0
\]

where \( i = 1, \ldots, m \). To obtain a non linear system of equations in the unknowns design
parameters \( \tau_{i} \), it is supposed that an approximated solution of this system is available \( \tau^{(0)} = (\tau_{1}^{(0)}, \ldots, \tau_{m}^{(0)}) \), such that in first approximation (6):

\[
T_{c}^{k} = G_{k}(\tau) \cong G_{k}(\tau^{(0)}) + \sum_{i=1}^{m} \frac{\partial G_{k}}{\partial \tau_{i}} \Delta \tau_{i}^{(1)}
\]

where \( \Delta \tau_{j} = \tau_{j}^{(1)} - \tau_{j}^{(0)} \). Contracting the equations above, it can be written:
\[
\sum_{k=1}^{n} \left[ T_k^0 - G_k(\tau^{(0)}) \right] - \sum_{j=1}^{m} \frac{\partial G_k}{\partial \tau_j} \Delta \tau_j \frac{\partial G_k}{\partial \tau_i} = 0
\] 

(7)

where \(i=1\ldots m\) or after changing the summation order and rearranging terms:

\[
\sum_{j=1}^{m} \left[ \sum_{k=1}^{n} \frac{\partial G_k}{\partial \tau_j} \frac{\partial G_k}{\partial \tau_i} \right] \Delta \tau_j = \sum_{k=1}^{n} \left[ T_k^0 - G_k(\tau^{(0)}) \right] \frac{\partial G_k}{\partial \tau_i}
\]

(8)

Expressions are the normal equations of the optimization problem:

\[
A^{(1)} \Delta \tau^{(1)} = b
\]

(9)

The matrix elements of this linear system are calculated with:

\[
A_{ij}^{(1)} = \sum_{k=1}^{n} \frac{\partial G_k}{\partial \tau_i} \frac{\partial G_k}{\partial \tau_j}
\]

(10)

where \((i=1\ldots m \text{ és } j=1\ldots m)\) and the components of the independent term applying:

\[
b_{ij}^{(1)} = \sum_{k=1}^{n} \left[ T_k^0 - G_k(\tau^{(0)}) \right] \frac{\partial G_k}{\partial \tau_i}
\]

(11)

where \(i=1\ldots m\). The derivatives (sensitivity coefficients) \(\frac{\partial G_k}{\partial \tau_j}\) \((k=1,\ldots, n)\) can be evaluated numerically and a central difference scheme is adopted here:

\[
\frac{\partial G_k}{\partial \tau_i} \approx \frac{G_k(\tau_i^0 + \varepsilon) - G_k(\tau_i^0 - \varepsilon)}{2\varepsilon}
\]

(12)

where the \(G_k(\tau_i^0 \pm \varepsilon)\) are the calculated temperatures at each point \(r_k\), increasing or decreasing the coefficient \(\tau_i^{(0)}\) in a small quantity \(\varepsilon\). After solving the linear system, an updated approximation to the optimization problem is obtained:

\[
\tau_i^{(1)} = \tau_i^{(0)} + \Delta \tau_i^{(1)}
\]

(13)

On making use of \(\tau_i^{(1)}\) \((i=1,2,\ldots, m)\), an improved approximation can be obtained \(\tau_i^{(2)}\) \((i=1,2,\ldots, m)\) by solving a new linear system:

\[
A^{(2)} \Delta \tau_i^{(2)} = b^{(2)}
\]

(14)

\[
\tau^{(2)} = \tau^{(1)} + \Delta \tau^{(2)}
\]

(15)
This iterative procedure (Fig 3.) is repeated until corrections $\Delta \tau_i^{(k)}$ between measured and estimated values of temperatures, satisfy certain convergence criterion (Eq. 4) $S < \eta$, where $\eta$ is a small quantity.

![Figure 3. The flowchart of the inverse algorithm](image)

### 3. APPLICATION

The inverse numerical method detailed above was implemented in the software SQintegra [3,4]. This program is used as the evaluation tool of the ivfSmartQuench instrument [5]. The following data are requested to estimate the HTC(T): the cooling curve represented by time-temperature data-pairs, the geometry and dimensions of the workpiece investigated, the location of the thermocouples within the workpiece, the specific heat and conductivity of the workpiece’s material and the temperature of the medium (i.e. the quenchant) surrounding of the piece.

![Figure 4. The cooling curve obtained in oil](image)

![Figure 5. The value of S (Eq. 4) as a function of iterations](image)
The following example illustrates the applicability of the algorithm developed. A cooling curve has been recorded in an oil based quenchant (temperature of the medium was 50 °C) by using the JIS K 2242 silver probe (Fig 4.). The geometry and the thermophysical parameters of the JIS probe was taken into consideration for the calculations [6]. The thermocouple is located at surface of the probe.

Performing 10 iterations the value of S was convergated to zero which resulted small the difference between the measured and the calculated cooling curves (Fig 5.). The maximum of the deviation between these two curves was 1.5 °C. The estimated Heat Transfer Coefficient function is illustrated on Fig 7. Parallel to the iterative calculations the HTC(T) was approximated by Lumped capacitance method as well [7] using the same cooling curve and input parameters. It can be concluded that the calculations performed by the different inverse methods derived almost identical results.

4. CONCLUSIONS

An iterative regularization method has been developed for estimation of the Heat Transfer Coefficient obtained during quenching as a function of surface temperature. The inverse algorithm is embedded into the SQIntegra software. A case study is carried out to calculate the HTC as a function of surface temperature using the cooling curve recorded by JIS silver probe. The HTC(T) functions has been calculated by the Lumped capacitance manner and the inverse algorithm. The results of the comparison study has demonstrated the reliability of the predicted HTC functions and the applicability of the numerical inverse method developed.

ACKNOWLEDGMENTS

The authors would like to acknowledge for the financial support from NKTH to this investigation under the Bilateral Cooperation Program Argentina/Hungary (ARG-11/2006), Mexico/Hungary (MEX-12/2007) and Slovenia/Hungary (SI-12/2008).
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